Int. J. Solids Structures, 1972, Vol. 8, pp. 1339 to 1351. Pergamon Press. Printed in Great Britain

THE LOAD CARRYING CAPACITIES OF SYMMETRICALLY LOADED SHALLOW SHELLS

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Abstract—A rigid, perfectly plastic analysis is presented herein for a shallow shell of degree $n \geq 2$ subjected to a uniformly distributed static pressure. Exact theoretical results have been obtained for three approximate yield surfaces when the outer boundary of a shell of second degree is either simply supported or fully clamped.

NOTATION

e_θ, e_ϕ	membrane strains in the θ and ϕ directions, respectively
h	$H/2R$
k_θ, k_ϕ	curvature changes in the θ and ϕ directions, respectively
m_θ, m_ϕ	M_θ/M_0 and M_ϕ/M_0, respectively
n	degree of shallow shell defined by equation (11)
n_θ, n_ϕ	N_θ/N_0 and N_ϕ/N_0, respectively
p_n, p_ϕ, p_c, p_s	RP_n/N_0, RP_ϕ/N_0, RP_c/N_0 and RP_s/N_0, respectively
r	coordinate defined in Fig. 1
s	S/N_0
t	time
u_n, u_ϕ	displacements defined in Fig. 1
v, w	u_ϕ/R and u_n/R, respectively
x, y	r/R and z/R, respectively
z	coordinate defined in Fig. 1
$2H$	shell thickness
M_0	$\sigma_0 H^2$
M_θ, M_ϕ	bending moments defined in Fig. 2
N_0	$2\sigma_0 H$
N_θ, N_ϕ	membrane forces defined in Fig. 2
P_n, P_ϕ	external pressures defined in Fig. 2
P_c, P_s	static collapse pressures
R	base radius of a shallow shell as shown in Fig. 1
S	transverse shear force defined in Fig. 2
T_0	reference time
Z	total depth of a shallow shell
γ	$\dfrac{\rho R^2}{N_0 T_0^2}$
θ, ϕ	coordinates of a shallow shell defined in Fig. 2
$\kappa_\theta, \kappa_\phi$	$\dfrac{M_0 k_\theta}{N_0}$ and $\dfrac{M_0 k_\phi}{N_0}$, respectively
ρ	surface density of the shell material
σ_0	tensile yield stress
τ	t/T_0
$[X]$	difference between values of X on either side of a circular line

$(\dot{\ })$ $\quad \frac{\partial}{\partial \tau}$

$(\ )'$ $\quad \frac{\partial}{\partial x}(\ )$

INTRODUCTION

PLASTIC methods of structural design are fairly well established and appear to be useful in a wide variety of situations [1–3, etc.]. Hodge and Lakshmikantham [4, 5] investigated the static behavior of shallow shells pierced with a central circular hole and made from a rigid, perfectly plastic material. The authors used the two-moment limited interaction yield condition and obtained upper and lower bounds on an actual limit pressure which would be distributed uniformly over the entire surface area. Theoretical results were presented for shells which were simply supported around the outer boundary.

Mróz and Bing-Ye [6] have studied the load carrying capacities of various axisymmetrically loaded spherical shells while Duszek [7] has examined the influence of finite-deflections on the behavior of a simply supported shallow spherical shell. Recently Biron and Chawla [8] have developed a numerical procedure in order to obtain upper and lower bounds for shells of revolution of arbitrary shape under general axisymmetric loading.

A rigid, perfectly plastic analysis is presented herein for a shallow shell of degree $n \geq 2$ which is subjected to a uniformly distributed static pressure. Exact theoretical solutions are obtained for three approximate yield surfaces when the outer boundary of a shell with $n = 2$ is either simply supported or fully clamped. The theoretical procedure is relatively simple and will be employed in a study of the dynamic response of shallow shells to be considered in another article [9].

BASIC EQUATIONS

It may be shown [1] that the generalized strain rates for rotationally symmetric shallow shells undergoing infinitesimal deflections and loaded symmetrically are

$$\dot{e}_\theta = \frac{\dot{v} - y'\dot{w}}{x} \tag{1a}$$

$$\dot{\kappa}_\theta = \frac{-h}{x}(\dot{w}' + y''\dot{v}) \tag{1b}$$

$$\dot{e}_\phi = \dot{v}' - y''\dot{w} \tag{1c}$$

and

$$\dot{\kappa}_\phi = -h(\dot{w}' + y''\dot{v})' \tag{1d}$$

when

$$(y')^2_{\max} \ll 1$$

and

$$h = \frac{H}{2R}. \tag{2}$$

The various quantities which appear in (1a)–(1d) are defined in Fig. 1 and the Notation. The principle of virtual velocities may be used in order to derive the following set of equations of motion which are consistent with (1a)–(1d)

$$xs = h[(xm_\phi)' - m_\theta] + \tfrac{4}{3}\gamma h^2 x\Omega_\phi \tag{3a}$$

$$(xn_\phi)' - n_\theta - y''xs + xp_\phi = x\gamma\ddot{v} \tag{3b}$$

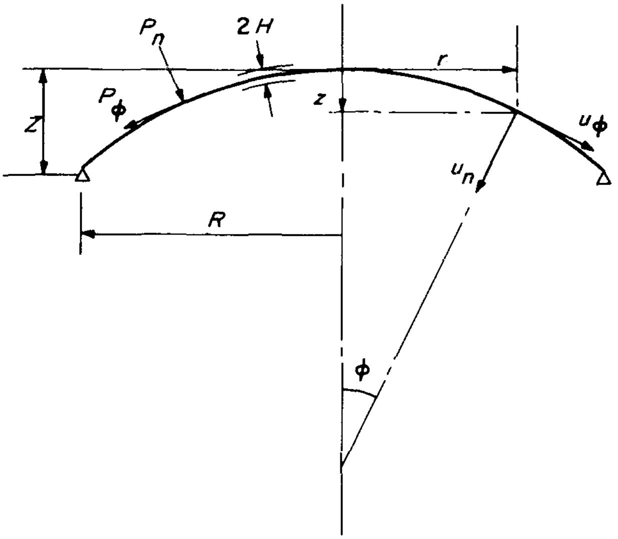

FIG. 1. Shallow shell.

and

$$y''xn_\phi + y'n_\theta + (xs)' + xp_n = x\gamma\ddot{w} \tag{3c}$$

where $\dot{\Omega}_\phi = \dot{w}' + y''\dot{v}$ and the various symbols are defined in the Notation and Fig. 2.

Onat and Prager [10] derived a four-dimensional yield surface for a rotationally symmetric shell made from a rigid, perfectly plastic material which obeys the Tresca yield condition. However, in order to make this general class of problems tractable it is usually necessary to simplify the yield surface [1]. The particular yield surfaces which are employed in the current investigation are shown in Fig. 3. It may be shown that the yield surfaces indicated in Figs. 3(a) and (b) completely circumscribe the yield surface derived by Onat and Prager [10] while others respectively 0·618 and 0·309 times as large would inscribe it. Yield surfaces 2 and 0·618 times as large as that drawn in Fig. 3(c) would circumscribe and inscribe the Tresca yield surface, respectively. However, if a solution to a particular problem utilized only a portion of any of these yield surfaces then the bounds on the limit load might be closer than the bounds quoted above.

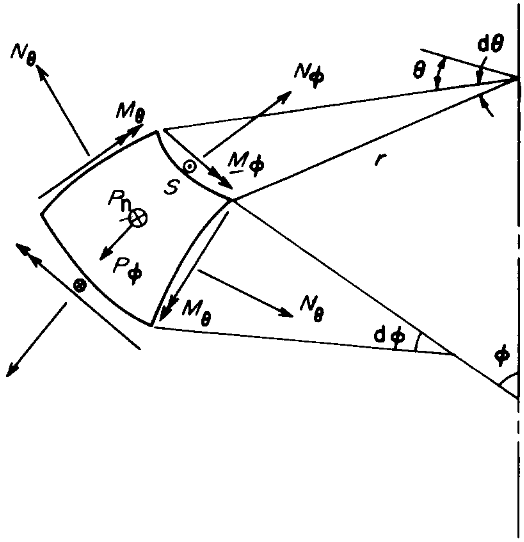

FIG. 2. Stress resultants for a shallow shell.

STATIC COLLAPSE PRESSURE OF A SIMPLY SUPPORTED SHALLOW SHELL

A cursory study of the title problem suggests that a solution should lie on the portions 4–7 or 5–7 of the two-moment limited interaction yield surface illustrated in Fig. 3(a). If regime 5–7 is selected then

$$n_\phi = -1, \qquad -1 \leq n_\theta \leq 0, \qquad \dot{e}_\theta = 0 \quad \text{and} \quad \dot{e}_\phi \leq 0 \tag{4a}$$

and

$$m_\theta = 1, \qquad 0 \leq m_\phi \leq 1, \qquad \dot{\kappa}_\phi = 0 \quad \text{and} \quad \dot{\kappa}_\theta \geq 0. \tag{4b}$$

Thus substituting equations (4a) and (4b) into equations (3a)–(3c) which are specialized to the static case gives

$$xs = h[(xm_\phi)' - 1] \tag{5a}$$

$$-1 - n_\theta - y''xs = 0 \tag{5b}$$

and

$$-y''x + y'n_\theta + (xs)' + xp = 0 \tag{5c}$$

when $p_\phi = 0$ and $p_n = p$. The solution of equations (5b) and (5c) is

$$xs = -\frac{px^2}{2} + xy' \tag{6a}$$

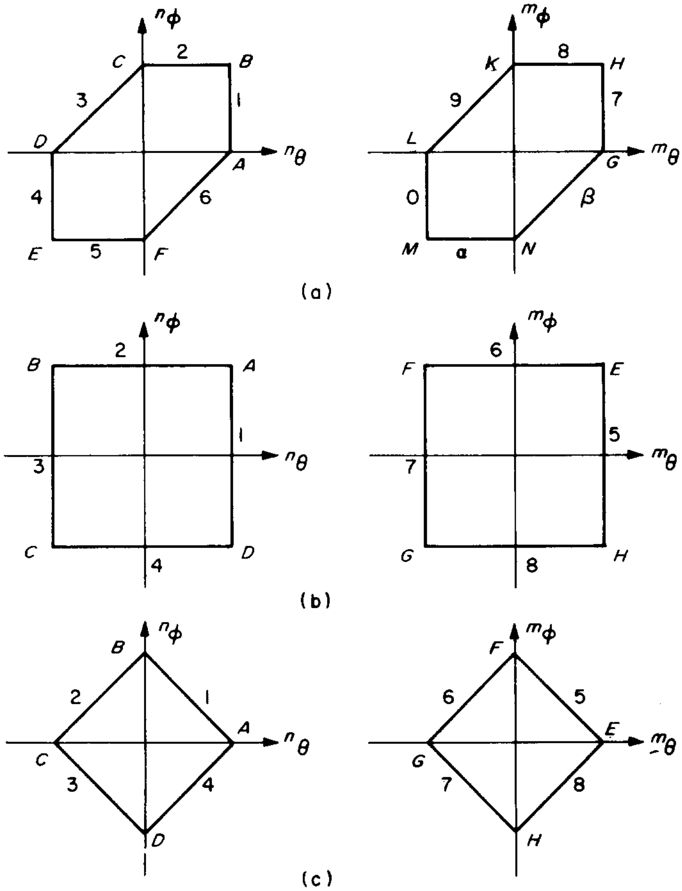

FIG. 3. (a) Two moment limited interaction yield condition. (b) Uncoupled square yield condition. (c) Uncoupled diamond yield condition.

and

$$n_\theta = -1+y''\left(\frac{px^2}{2}-xy'\right) \tag{6b}$$

provided $(y')^2 \ll 1$.

If (6a) is substituted into (5a) then it may be shown that

$$hxm_\phi = hx-\frac{px^3}{6}+\int_0^x \xi y' \,\mathrm{d}\xi \tag{7}$$

which gives the static collapse pressure

$$p_s = 6h+6\int_0^1 \xi y' \,\mathrm{d}\xi \tag{8}$$

from the requirement that $m_\phi = 0$ at $x = 1$. It might be remarked in passing that (8) gives $p_s = 6h$ when $y' = 0$ which agrees with the predictions of Hopkins and Prager [11] for a uniformly loaded flat circular plate.

The stress resultants (6a), (6b) and (7) may be written in the form

$$xs = -3x^2\left(h+\int_0^1 \xi y'\,\mathrm{d}\xi\right)+xy' \tag{9a}$$

$$n_\theta = -1+y''\left[3x^2\left(h+\int_0^1 \xi y'\,\mathrm{d}\xi\right)-xy'\right] \tag{9b}$$

and

$$m_\phi = 1-x^2\left(1+\frac{1}{h}\int_0^1 \xi y'\,\mathrm{d}\xi\right)+\frac{1}{hx}\int_0^x \xi y'\,\mathrm{d}\xi. \tag{9c}$$

Now the flow rule associated with the regime 5–7 of the yield surface illustrated in Fig. 3(a) demands $\dot{e}_\theta = \dot{\kappa}_\phi = 0$. Thus,

$$\dot{v} = y'\dot{w} \tag{10a}$$

and

$$\dot{w}'+y''\dot{v} = \text{const.} \tag{10b}$$

which have the solutions

$$\dot{w} = \dot{w}_0(1-x) \tag{10c}$$

and

$$\dot{v} = y'\dot{w}_0(1-x) \tag{10d}$$

when using the shallow shell approximation and the kinematical requirements at $x = 1$. It is straightforward to show for a shallow shell that (10c) and (10d) give $\dot{e}_\phi \leq 0$ and $\dot{\kappa}_\theta \geq 0$ which agrees with the normality requirements (4a) and (4b).

The equation of the middle surface of a shallow shell may be written in the form [1]

$$y = \frac{Z}{R}x^n \tag{11}$$

where Z is the total depth of a shell and $n > 1$ for a smooth surface. It should be noted that the analysis of a shallow shell presented above incorporates the restriction

$$(y'_{\max})^2 = \frac{n^2Z^2}{R^2} \ll 1. \tag{12}$$

Equations (8) and (9) may now be written

$$p_s = 6h+\frac{6nZ}{(n+1)R} \tag{13}$$

$$xs = -3x^2\left(h+\frac{nZ}{(n+1)R}\right)+\frac{nZx^n}{R} \tag{14a}$$

$$n_\theta = -1+n(n-1)\frac{Z}{R}\left[3\left(h+\frac{nZ}{(n+1)R}\right)x^n-\frac{nZx^{2(n-1)}}{R}\right] \tag{14b}$$

and

$$m_\phi = 1 - x^2 + \frac{nZ}{(n+1)hR}(x^n - x^2). \tag{14c}$$

It may be shown that $0 \leq m_\phi \leq 1$ provided

$$\frac{n(n-2)Z}{(n+1)hR} < 2 \tag{15a}$$

and

$$n \geq 2. \tag{15b}$$

Moreover, $n_{\theta'} \geq 0$ throughout $0 \leq x \leq 1$ provided

$$\frac{3n(n-2)Z}{2R}\left(h + \frac{Z}{R}\right) < 1$$

which is satisfied for a shallow shell with a practical value of n. The requirement that $-1 \leq n_\theta \leq 0$ at $x = 1$ imposes no additional restrictions. Thus the limit pressure (13) for a shallow shell which has a middle surface described by (11) is kinematically and statically admissible and is therefore exact for the two-moment limited interaction yield surface provided the inequalities (15a) and (15b) are satisfied.

The exact static collapse pressure for a shallow shell with a surface of second degree ($n = 2$), which may be a shallow spherical, paraboloidal, ellipsoidal or hyperboloidal cap, is

$$p_s = 6h + \frac{4Z}{R} \tag{16}$$

since equations (15a) and (15b) are always satisfied. However, the actual collapse pressure (p_c) is given by

$$0{\cdot}618 p_s \leq p_c \leq p_s \tag{17}$$

since the two-moment limited interaction yield surface circumscribes the Tresca yield surface [10] while another 0·618 times as large would inscribe it. It may be shown that the predictions of Duszek [7] for the static collapse pressure of a shallow shell with $(Z/R)^2 \ll 1$ is

$$p_s = 6h + \frac{16hZ^2}{5H^2} \tag{18}$$

which lies between the bounds (17) provided $Z/H \leq 2{\cdot}5$. It should be noted that the analysis in Ref. [7] is developed for a vertical pressure loading which acts in the z direction whereas equations (16) and (17) are based on a uniformly distributed pressure which is normal to the middle surface of a shallow shell.

The foregoing analysis may be repeated for a shallow shell made from a material which obeys the uncoupled square yield condition shown in Fig. 3(b). The only difference from the previous work is that the minimum value of m_ϕ may now be -1 which gives

$$\frac{n(n-2)Z}{(n+1)R} \leq (n+2)h \tag{19}$$

as the condition for static admissibility.

The uncoupled diamond yield surface illustrated in Fig. 3(c) simplifies considerably the theoretical analysis of a shallow shell but at the expense of more disparate bounds. However, the upper bound is actually lower and therefore better than that predicted by equation (16). If the faces 3 and 5 of this yield surface are selected then it may be shown for a shallow shell of second degree that the corresponding collapse pressure is

$$p_s = 4h+\frac{2Z}{R} \tag{20}$$

with an associated generalized stress field

$$n_\theta = -\tfrac{1}{2}+\frac{hZx^2}{R} \tag{21a}$$

$$n_\phi = -\tfrac{1}{2}-\frac{hZx^2}{R} \tag{21b}$$

$$m_\theta = \tfrac{1}{2}(1+x^2) \tag{21c}$$

and

$$m_\phi = \tfrac{1}{2}(1-x^2) \tag{21d}$$

which is statically admissible because h and Z/R are small quantities. Now the kinematically admissible velocity field

$$\dot{v} = 0 \tag{22a}$$

and

$$\dot{w} = \dot{w}_0(1-x^2) \tag{22b}$$

gives generalized strain rates which are normal to the corresponding portion of the yield surface. Thus, according to the limit theorems of plasticity, equation (20) is the exact collapse pressure for the uncoupled diamond yield surface.

Equations (21a) and (21b) give $n_\theta = n_\phi = -1/2$ for a shallow shell. It can be shown, therefore, that it is possible to circumscribe and inscribe the portion of the Tresca yield surface [10] appropriate to the problem at hand with surfaces which are respectively 1·5 and 0·75 times as large as that drawn in Fig. 3(c). Thus, the actual collapse pressure (p_c) is given by

$$0{\cdot}75p_s \le p_c \le 1{\cdot}5p_s. \tag{23}$$

The various predictions for the static collapse pressure of a simply supported shallow shell of second degree are compared in Fig. 4.

STATIC COLLAPSE PRESSURE OF A FULLY CLAMPED SHALLOW SHELL

A preliminary study of this particular problem indicates that it would be convenient to divide a shallow shell into the two zones $0 \le x \le u$ and $u \le x \le 1$. It is evident that the regimes 5–7 and $5'$–β of the yield surface illustrated in Fig. 3(a) could be used to describe the behavior in these two regions. The theoretical procedure for the inner zone $0 \le x \le u$ is similar to that for the simply supported case considered earlier. Thus equations (6a),

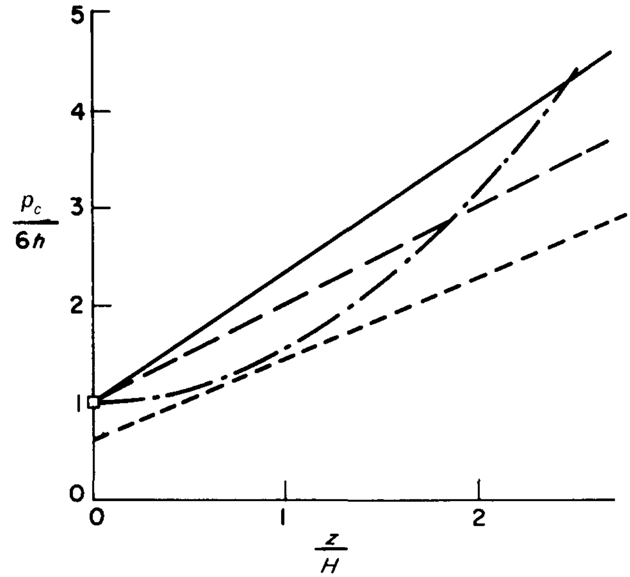

FIG. 4. Comparison of theoretical predictions for a simply supported shallow shell of second degree: □, Hopkins and Prager [11] solution for $Z/H = 0$; ———, equation (16), upper bound using yield condition shown in Fig. 3(a); — — — — —, 0·618X equation (16), lower bound; —·—·—, equation (18) which was predicted by Duszek [7]; - - - -, 1·5X equation (20), upper bound using yield condition shown in Fig. 3(c).

(6b) and (7) for a shallow shell of second degree give

$$xs = -\frac{x^2}{2}\left(p-\frac{4Z}{R}\right) \tag{24a}$$

$$n_\theta = -1+\frac{x^2Z}{R}\left(p-\frac{4Z}{R}\right) \tag{24b}$$

and

$$m_\phi = 1-\frac{x^2}{6h}\left(p-\frac{4Z}{R}\right) \tag{24c}$$

while the corresponding velocity field from (10a) and (10b) is

$$\dot{w} = \dot{w}_0+Ax \tag{25a}$$

and

$$\dot{v} = \frac{2xZ}{R}(\dot{w}_0+Ax). \tag{25b}$$

In the outer zone $u \leq x \leq 1$ the yield condition for the regime 5–β and equations (3a)–(3c) may be shown to give equations (24a) and (25b) and

$$m_\phi = \log\left(\frac{x}{u}\right)-\frac{(x^2-u^2)}{4h}\left(p-\frac{4Z}{R}\right) \tag{26}$$

since $m_\phi(u^+) = 0$. The flow rule associated with this portion of the yield surface requires $\dot{e}_\theta = 0$ and $\dot{\kappa}_\theta + \dot{\kappa}_\phi = 0$, or

$$\dot{w} = B \log x \tag{27a}$$

and

$$\dot{v} = \frac{2xZB}{R} \log x. \tag{27b}$$

If the four unknowns A, B, p and u which appear in equations (24a)–(27b) are determined from the four conditions $m_\phi(u^-) = 0$, $m_\phi(1) = -1$, $[\dot{w}] = 0$ at $x = u$ and $[\dot{w}'] = 0$ at $x = u$ then it may be shown that the static collapse pressure is

$$p_s = \frac{6h}{u^2} + \frac{4Z}{R} \tag{28a}$$

where

$$\frac{3}{u^2} = 5 - \log u^2 \tag{28b}$$

or

$$u \simeq 0{\cdot}731 \tag{28c}$$

and that the velocity field (25a), (25b), (27a) and (27b) can be written in the form

$$\dot{w} = \dot{w}_0 \left[1 - \frac{x}{u(1-\log u)} \right], \qquad 0 \le x \le u \tag{29a}$$

$$\dot{v} = \frac{2Zx\dot{w}_0}{R} \left[1 - \frac{x}{u(1-\log u)} \right], \qquad 0 \le x \le u \tag{29b}$$

$$\dot{w} = -\dot{w}_0 \frac{\log x}{1-\log u}, \qquad u \le x \le 1 \tag{29c}$$

and

$$\dot{v} = -\frac{2Zx\dot{w}_0 \log x}{R(1-\log u)}, \qquad u \le x \le 1. \tag{29d}$$

This velocity field is kinematically admissible since the generalized strain rates given by equations (1a)–(1d) satisfy the normality requirements of plasticity when invoking the shallow shell approximation. Thus the static collapse pressure (p_s) predicted by equation (28a) is exact according to the limit theorems because the generalized stresses given by substituting (28a) into equations (24b), (24c) and (26) are statically admissible. Again the actual collapse pressure (p_c) is bounded as indicated by (17). Equations (28a) and (28c) predict $p_s = 11{\cdot}26h$ when $Z/R \to 0$ which agrees with the exact theoretical solution obtained by Hopkins and Prager [11] for a fully clamped circular plate made from a rigid, perfectly plastic material which obeys the Tresca yield criterion.

If a shallow shell of second degree is made from a material which obeys the uncoupled square yield surface illustrated in Fig. 3(b), then it may be shown that equations (7) and (11) give a static collapse pressure

$$p_s = 12h+\frac{4Z}{R} \tag{30}$$

from the boundary conditions $m_\phi = -1$ at $x = 1$. It may be shown that the associated generalized stress field is statically admissible while the velocity field (10c) and (10d) is kinematically admissible. Thus p_s given by equation (30) is an exact collapse pressure for the uncoupled square yield surface.

A fully clamped shallow shell of second degree which obeys the uncoupled diamond yield criterion drawn in Fig. 3(c) collapses into two distinct zones $0 \le x \le u$ and $u \le x \le 1$. The regime 3–5 of Fig. 3(c) governs the behavior in the inner zone $0 \le x \le u$ while the portion 3–8 controls the response in the outer region $u \le x \le 1$. It is evident that an analysis of the inner zone $0 \le x \le u$ is similar to that for the simply supported case studied earlier. Thus,

$$s = -\frac{x}{2}\left(p-\frac{2Z}{R}\right) \tag{31a}$$

$$n_\phi = -\frac{1}{2}-\frac{x^2Z}{4R}\left(p-\frac{2Z}{R}\right) \tag{31b}$$

$$n_\theta = -\frac{1}{2}+\frac{x^2Z}{4R}\left(p-\frac{2Z}{R}\right) \tag{31c}$$

$$m_\phi = \frac{1}{2}-\frac{x^2}{8h}\left(p-\frac{2Z}{R}\right) \tag{31d}$$

$$m_\theta = \frac{1}{2}+\frac{x^2}{8h}\left(p-\frac{2Z}{R}\right) \tag{31e}$$

$$\dot{v} = 0 \tag{32a}$$

and

$$\dot{w} = \dot{w}_0+Bx^2. \tag{32b}$$

The equilibrium equations (3a)–(3c) may be solved with the aid of the yield condition for face 3–8 to give

$$m_\phi = \log\left(\frac{x}{u}\right)-\frac{(x^2-u^2)}{4h}\left(p-\frac{2Z}{R}\right) \tag{33}$$

since $m_\phi(u^+) = 0$. It may be shown that the flow rule and kinematical conditions require

$$\dot{v} = 0 \tag{34a}$$

and

$$\dot{w} = C\log x. \tag{34b}$$

The four unknowns p, u, B and C which appear in equations (31a)–(34b) may be found from the requirements $[m_\phi] = 0$, $[\dot{w}] = 0$ and $[\dot{w}'] = 0$ at $x = u$ and $m_\phi = -1$ at $x = 1$.

Finally it may be shown that the static collapse pressure is

$$p_s = \frac{4h}{u^2} + \frac{2Z}{R} \tag{35a}$$

where

$$\frac{1}{u^2} = 2 - \log u \tag{35b}$$

or

$$u \simeq 0{\cdot}64. \tag{35c}$$

It may be verified that equation (35a) is an exact collapse pressure for the uncoupled diamond yield condition.

The various predictions for the static collapse pressure of a fully clamped shallow shell of second degree are compared in Fig. 5.

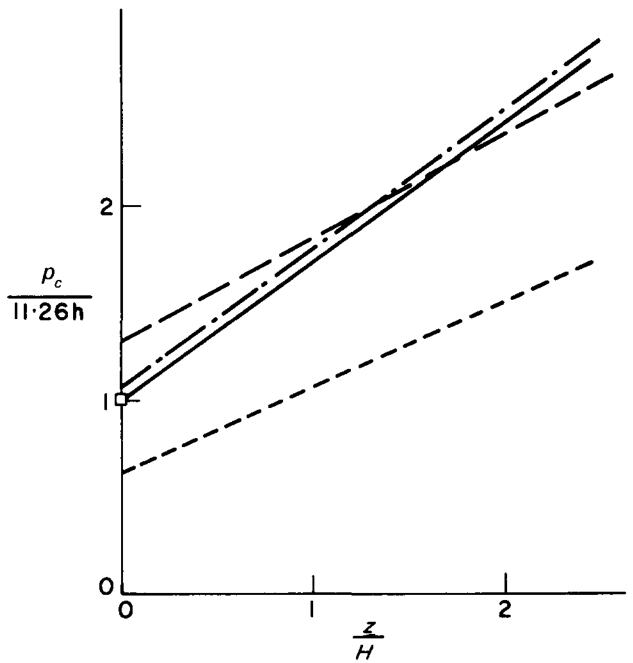

FIG. 5. Comparison of theoretical predictions for a fully clamped shallow shell of second degree: □, Hopkins and Prager [11] solution for $Z/H = 0$; ————, equation (28a), upper bound using yield condition shown in Fig. 3(a); – – – – –, 0·618X equation (28a), lower bound; — · — · —, equation (30), upper bound using yield condition shown in Fig. 3(b); — — — —, 1·5X equation (35a), upper bound using yield condition shown in Fig. 3(c).

CONCLUSIONS

A rigid, perfectly plastic analysis is presented herein for a shallow shell of degree $n \geq 2$ subjected to a uniformly distributed static pressure. Exact theoretical results have been obtained for three approximate yield surfaces when the outer boundary of a shell of second degree is either simply supported or fully clamped. The analytical procedure is relatively simple and will be employed in a study of the dynamic response of shallow shells to be considered in another article [9].

Acknowledgement—The authors wish to thank the Engineering Mechanics Program of the National Science Foundation for their support of this work under contract number GK-20189X.

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(*Received* 27 *December* 1971)

Абстракт—В рамках жесткой идеальной пластичности дается анализ для пологой оболочки порядка и ≥ 2, подверженной действию равномерно распределенного статистического давления. Получаются точные теоретические результаты для мрех прцбиц нсеререых поверхносмей мечеицъ, когда внешний край оболочки, порядка 2, свободно опертый либо полностью защемлен. Теоретический анализ оказывается относительно несложен. Он также пригодный для исследования динамического поведения пологих оболочек, которое будет рассматриваться в другой работе [9].